

# Stability of the Anisotropic Brane Cosmology

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The stability of the Bianchi type I anisotropic brane cosmology is analyzed in this paper. We also study the effect of the brane solution by comparing the models on the 3-brane and the models in the conventional Einstein's space. Analysis is presented for two different models: one with a perfect fluid and the other one with a dilaton field. It is shown that the anisotropic expansion is smeared out dynamically for both theories in the large time limit independent of the models with different types of matter. The initial states are, however, dramatically different. A primordial anisotropic expansion will grow for the conventional Einstein's theory. On the other hand, it is shown that the initial state is highly isotropic for the brane universe except for a very particular case. Moreover, it is also shown that the Bianchi type I anisotropic cosmology is stable against any anisotropic perturbation for both theories in the large time limit.

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## I. INTRODUCTION

The observation of the cosmic microwave background (CMB) radiation [1, 2] indicates that our Universe is globally isotropic to a very high degree of precision. Therefore, our Universe is usually assumed to be described by the Friedmann-Robertson-Walker (FRW) metric in most of the literatures. The origin of the isotropic universe has also become an interesting research topic ever since.

On the other hand, it is known that there is a small large-angle CMB anisotropies,  $\Delta T/T \simeq 10^{-5}$ , under the CMB background [1, 2]. The isotropy of our Universe may have to do with the choice of initial conditions and the stability of the evolutionary solutions. Therefore, the present isotropic phase could be a dynamic result of the evolution of our Universe, no matter what the initial state started out. In this paper, we will address on this issue in both the standard Einstein's theory and the newly developed brane world scenario [3, 4]. In particular, we will consider the evolution of an anisotropic cosmology described by the Bianchi type I models [5]-[12] for two different types of matter sources: one is the perfect fluid and the other one is a dilaton field. Our result shows that the anisotropic models considered all evolve dynamically from the anisotropic Bianchi type I universe into the isotropic FRW space in the large time limit. This property reveals that the isotropy of the cosmological principle may be justified and made consistent with our current observational data.

Those two different theories give, however, completely different initial anisotropic expansion at the very early stage of evolution. Indeed, for the conventional Einstein's theory (CET), the anisotropic expansion tends to be large in the very early stage. In another words, the universe has to begin from a highly anisotropic initial expansion and then decays to zero as the time increases. On the other hand, in the brane world scenario [13]-[16], due to the quadratic correction which significantly changes the early time behavior of the Universe. As a result, any initially non-vanishing anisotropy parameter tends to vanish in the very early period. There is a characteristic time,  $t_c$ , that divides the evolution of mean anisotropy parameter into two different stages. The mean anisotropy parameter is increasing when  $t < t_c$  and reaches its maximal value at  $t = t_c$ . After that, mean anisotropy parameter starts to decay. This result remains true for both the model with a perfect fluid and the model with a scalar field. And this appears to be a general feature independent of the types of matter considered.

In addition, the stability analysis [17]-[24] indicates that all models in two different theories are all stable against any anisotropic perturbation. In particular, we also show that the system is stable against the dilaton perturbation when the dilaton potential is close to its local minimum. Note that the exact solution is known only when the dilaton potential is a constant. This also justifies the stability of the isotropic universe in the large time limit under the effect

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of a dilaton potential with local minima.

This paper is organized as follows. In Sec. II we briefly review the brane world formulation and the Bianchi type I model. Then we consider the evolution of the anisotropic cosmology and its stability in both theories of the CET and the brane world. Analysis is presented for model coupled to a perfect fluid in section III. The model with a dilaton field and a constant potential is presented in section IV. Finally, the conclusion and possible implications are drawn in Sec. V.

## II. PRELIMINARY OF BRANE WORLD AND ANISOTROPIC COSMOLOGY

The brane world scenario assumes that our Universe is a four-dimensional space-time, a 3-brane, embedded in the 5D bulk space-time. All the matter fields and the gauge fields except the graviton are confined on the 3-brane as a prior requirement in order to avoid any violations with the empirical results. Moreover, inspired by the string theory/M-theory, the  $Z_2$ -symmetry with respect to the brane is imposed [27]. A formal realization of the brane world scenario, which recovers the Newton gravity in the linear theory, is the Randall-Sundrum model [3, 4] in which the 4D flat brane(s) is embedded in the 5D anti-de Sitter (AdS) space-time. Later on, a covariant formulation of the effective gravitational field equations on the 3-brane has been obtained via a geometric approach by Shiromizu, Maeda and Sasaki [13, 15]. It is shown that the effective four-dimensional gravitational field equations on the brane take the following form

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k_4^2 T_{\mu\nu} + k_5^4 S_{\mu\nu} - E_{\mu\nu}, \quad (1)$$

where  $G_{\mu\nu}$  and  $T_{\mu\nu}$  are the Einstein and energy-momentum tensors.  $S_{\mu\nu}$  is a quadratic contribution of  $T_{\mu\nu}$  defined as

$$S_{\mu\nu} = \frac{1}{12} T T_{\mu\nu} - \frac{1}{4} T_\mu^\alpha T_{\nu\alpha} + \frac{1}{24} g_{\mu\nu} (3T^{\alpha\beta} T_{\alpha\beta} - T^2). \quad (2)$$

The effective 4D parameters, e.g., the cosmological constant  $\Lambda$  and gravitational coupling  $k_4$ , are determined by the 5D bulk cosmological constant  $\Lambda_5$ , the 5D gravitational coupling  $k_5$  and the tension of the brane  $\lambda$  via the following relations

$$\Lambda = k_5^2 \left( \frac{\Lambda_5}{2} + \frac{k_5^2 \lambda^2}{12} \right), \quad k_4^2 = \frac{k_5^4 \lambda}{6}. \quad (3)$$

Here  $g_{\mu\nu}$  is the metric tensor on the brane. In addition, the quantity  $E_{\mu\nu}$  is a pure bulk effect defined by the bulk Weyl tensor [13].

From the Eq. (1), it is easy to realize that the brane world scenario is different from the CET by two parts: (a) the matter fields contribute local “quadratic” energy-momentum correction via the tensor  $S_{\mu\nu}$ , and (b) there are “nonlocal” effects from the free field in the bulk, transmitted via the projection of the bulk Weyl tensor  $E_{\mu\nu}$ . Therefore, the CET can be treated as a limit of the brane theory by taking  $E_{\mu\nu} = 0$  and  $k_5 \rightarrow 0$  with properly adjusted values of the constants  $k_4$  and  $\Lambda$ .

In addition to the generalized Einstein equations (1), the energy-momentum tensor also satisfies the conservation law  $\nabla_\mu T^{\mu\nu} = 0$ . Therefore, there is a constraint on the tensor  $E_{\mu\nu}$ ,  $\nabla_\mu E^{\mu\nu} = k_5^4 \nabla_\mu S^{\mu\nu}$ , due to the Bianchi identity on the brane. Here, the operator  $\nabla$  is the covariant derivative with respect to the metric  $g_{\mu\nu}$  on the brane. One should point out here that the field equations on the brane, namely the generalized Einstein equations, the conservation of energy-momentum and the constraint on  $E_{\mu\nu}$  are, in general, not a closed system in the 4D brane since the quantity  $E_{\mu\nu}$  is five-dimensional. It can only be evaluated by solving the field equations in the bulk. In this paper, we will, however, only consider the quadratic effect on the brane world in the anisotropic background. Therefore, we will set  $E_{\mu\nu} = 0$  which is equivalent to embedding the 3-brane in the pure AdS bulk space-time.

Another important subject in this paper is the anisotropic cosmology described by Bianchi type I metric. The line element of the Bianchi type I space, an anisotropic generalization of the flat FRW geometry, can be written as

$$ds^2 = -dt^2 + a_1^2(t) dx^2 + a_2^2(t) dy^2 + a_3^2(t) dz^2, \quad (4)$$

with  $a_i(t)$ ,  $i = 1, 2, 3$  the expansion factors on each different spatial directions. For later convenience, we will introduce the following variables

$$V \equiv \prod_{i=1}^3 a_i, \quad \text{volume scale factor}, \quad (5)$$

$$H_i \equiv \frac{\dot{a}_i}{a_i}, \quad i = 1, 2, 3, \quad \text{directional Hubble factors,} \quad (6)$$

$$H \equiv \frac{1}{3} \sum_{i=1}^3 H_i = \frac{\dot{V}}{3V}, \quad \text{mean Hubble factor.} \quad (7)$$

In addition, we will also introduce two basic physical observational quantities in cosmology:

$$A \equiv \sum_{i=1}^3 \frac{(H_i - H)^2}{3H^2}, \quad \text{mean anisotropy parameter,} \quad (8)$$

$$q \equiv \frac{d}{dt}(H^{-1}) - 1, \quad \text{deceleration parameter.} \quad (9)$$

Note that  $A \equiv 0$  for an isotropic expansion. Moreover, the sign of the deceleration parameter indicates how the Universe expands. Indeed, a positive sign corresponds to “standard” decelerating models whereas a negative sign indicates an accelerating expansion.

### III. ANISOTROPIC UNIVERSE WITH A PERFECT FLUID

In this section, we will consider the case that the matter energy-momentum tensor,  $T_{\mu\nu}$ , is a perfect fluid whose components are given by

$$T^0_0 = -\rho, \quad T^1_1 = T^2_2 = T^3_3 = p. \quad (10)$$

Here the energy density  $\rho$  and the pressure  $p$  of the cosmological fluid obey a linear barotropic equation of state of the form  $p = (\gamma - 1)\rho$  with  $\gamma$  a constant in the range  $1 \leq \gamma \leq 2$ .

#### A. Conventional Einstein's Theory

For the CET, the dynamics of the space-time is determined by the Einstein equations and the energy-momentum conservation law

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k_4^2 T_{\mu\nu}, \quad \nabla_\mu T^{\mu\nu} = 0, \quad (11)$$

with  $G_{\mu\nu}$  the Einstein tensor,  $\Lambda$  the cosmological constant and  $k_4$  the gravitational coupling  $k_4^2 = 8\pi G$ . For the Bianchi type I cosmology with a perfect fluid the  $tt$ - and  $ii$ -components of Einstein equations<sup>1</sup> and the conservation law become

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = \Lambda - \frac{3\gamma - 2}{2} k_4^2 \rho, \quad (12)$$

$$\frac{1}{V} \frac{d}{dt}(V H_i) = \Lambda - \frac{\gamma - 2}{2} k_4^2 \rho, \quad i = 1, 2, 3, \quad (13)$$

$$\dot{\rho} + 3\gamma H \rho = 0. \quad (14)$$

First of all, the Eq. (14) can be easily solved to obtain the time evolution law of the energy density of the fluid

$$\rho = \rho_0 V^{-\gamma}, \quad \rho_0 = \text{constant} > 0. \quad (15)$$

Next, by summing the Eqs. (13) with respect to the index  $i$ , we can obtain the relation

$$\frac{1}{V} \frac{d}{dt}(V H) = \Lambda - \frac{\gamma - 2}{2} k_4^2 \rho. \quad (16)$$

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<sup>1</sup> Actually, the equations (12) and (13) are the time and spatial components of the following version of the Einstein equation:  $R_{\mu\nu} = \Lambda g_{\mu\nu} + k_4^2 (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$ , with  $R_{\mu\nu}$  the Ricci tensor and  $T$  the trace of  $T_{\mu\nu}$ .

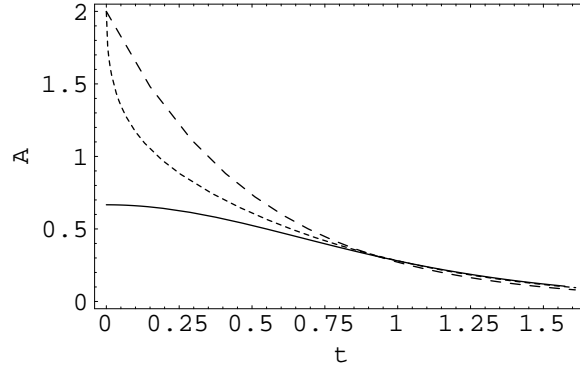


FIG. 1: Mean anisotropy parameter of the Bianchi type I conventional Einstein's theory with a perfect fluid:  $\gamma = 2$  (solid curve),  $\gamma = 1.5$  (dotted curve) and  $\gamma = 1$  (dashed curve). The normalization of the parameters is chosen as  $3\Lambda = 1$ ,  $3k_4^2\rho_0 = 2$ , and  $C = 1$ .

Subtracting this result with the Eqs. (13), we can show that

$$H_i = H + \frac{K_i}{V}, \quad i = 1, 2, 3, \quad (17)$$

with  $K_i$ ,  $i = 1, 2, 3$  the constants of integration satisfying the consistency condition  $\sum_{i=1}^3 K_i = 0$ . Moreover, by using the evolution law of the matter energy density (15), the basic equation (16) describing the dynamics of the anisotropic Universe can be written as

$$\ddot{V} = 3\Lambda V - \frac{3(\gamma - 2)}{2} k_4^2 \rho_0 V^{1-\gamma}, \quad (18)$$

with the general solution

$$t - t_0 = \int G(V)^{-1/2} dV, \quad (19)$$

where function  $G(V)$  is defined by

$$G(V) \equiv 3\Lambda V^2 + 3k_4^2 \rho_0 V^{2-\gamma} + C. \quad (20)$$

Here  $C$  are constants of integration. Based on this result, the other variables can be calculated straightforwardly and the answers are

$$H = \frac{G(V)^{1/2}}{3V}, \quad (21)$$

$$a_i = a_{0i} V^{1/3} \exp \left[ K_i \int V^{-1} G(V)^{-1/2} dV \right], \quad i = 1, 2, 3, \quad (22)$$

$$A = 3K^2 G(V)^{-1}, \quad (23)$$

$$q = \frac{18\gamma k_4^2 \rho_0 V^{2-\gamma} + 12C}{4G(V)} - 1. \quad (24)$$

Here the  $a_{0i}$ ,  $K_i$ ,  $i = 1, 2, 3$  are constants of integration and  $K^2 = \sum_{i=1}^3 K_i^2$ . In addition, the arbitrary integration constants  $K_i$  and  $C$  must satisfy the consistency condition

$$K^2 = \frac{2}{3}C. \quad (25)$$

Although the general solution can only be expressed in the parametric form via the volume scale factor  $V$ , the behaviors of the physical variables, namely the mean anisotropy parameter  $A$  and the deceleration parameter  $q$ , can be plotted with respect to the time as shown in Fig.1 and Fig.2.

Moreover, from the the above parametric expressions, we can still analyze the physics in two different interesting limits. First of all, let us consider the large  $t$  (or equivalently large  $V$ ) limit. Since the value of parameter  $\gamma$  is in

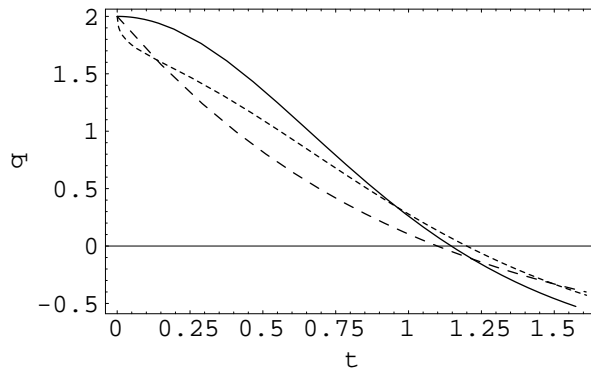


FIG. 2: Deceleration parameter of the Bianchi type I conventional Einstein's theory with a perfect fluid:  $\gamma = 2$  (solid curve),  $\gamma = 1.5$  (dotted curve) and  $\gamma = 1$  (dashed curve). The normalization of the parameters is chosen as  $3\Lambda = 1$ ,  $3k_4^2\rho_0 = 2$ , and  $C = 1$ .

the range  $1 \leq \gamma \leq 2$ , the asymptotic value of  $G(V)$  will approach  $G(V) \rightarrow 3\Lambda V^2$  when the cosmological constant is non-vanishing ( $\Lambda > 0$ ). This leads the volume scale parameter of our Universe to expand exponentially in the large time limit, i.e.  $V \propto \exp[\sqrt{3\Lambda}t]$ . Therefore, in this limit, the mean anisotropy parameter decays to zero exponentially,  $A \propto \exp[-2\sqrt{3\Lambda}t] \rightarrow 0$ , and the deceleration parameter becomes negative  $q \propto \exp[-\gamma\sqrt{3\Lambda}t] - 1 < 0$ . Hence the Universe can be isotropized dynamically and undergoes an accelerated expansion in the large time limit due to the presence of a positive cosmological constant  $\Lambda$ . In fact, the value of the cosmological constant can only change the expanding rate of the Universe but will not affect its effect of isotropization in general.

In order to illustrate this point, let us consider the model with a vanishing cosmological constant  $\Lambda = 0$ . Therefore, we have  $G(V) \propto V^{2-\gamma}$  in the large time limit. This leads to the results  $V \propto t^{2/\gamma}$ ,  $A \propto t^{2-4/\gamma}$ , and  $q \rightarrow 3\gamma/2 - 1 > 0$ . Hence, the Universe can still be isotropized dynamically except for the case when  $\gamma = 2$ . Note that the evolution will, however, be deaccelerated in the case  $\gamma = 2$ .

Next, let us focuss on the earlier stages of the above exact solutions. For simplicity, we will assume  $C = 0$  in this case. One can show that  $G(V) \sim V^{2-\gamma}$ . Therefore, one can solve for  $V \propto t^{2/\gamma}$  from the Eq. (21). Hence one has  $A \propto t^{2-4/\gamma}$ , and  $q \rightarrow 3\gamma/2 - 1 > 0$  from the rest of the field equations. Therefore, at the earlier period stage of the Universe, the evolution is decelerating even a cosmological constant is present. This is also shown numerically in the Fig.1 and Fig.2. Moreover, the mean anisotropy parameter  $A$  is always non-vanishing independent of the values of  $\gamma$ . This means that the early Universe is always anisotropic. Therefore, for the Universe with perfect fluid matter in the CET, the initial state is always anisotropic and this primordial anisotropy is smeared away as a consequence of the evolution of the Universe.

## B. Brane Cosmology

We will focuss on the brane effect for the model with a perfect fluid in this subsection. The same constraint for the perfect fluid conservation law (14) still holds on the brane. In addition, the gravitational field equations and the conservation law on the brane take the form, in terms of the variables (5)-(7),

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = \Lambda - \frac{3\gamma-2}{2} k_4^2 \rho - \frac{3\gamma-1}{12} k_5^4 \rho^2, \quad (26)$$

$$\frac{1}{V} \frac{d}{dt} (V H_i) = \Lambda - \frac{\gamma-2}{2} k_4^2 \rho - \frac{\gamma-1}{12} k_5^4 \rho^2, \quad i = 1, 2, 3, \quad (27)$$

$$\dot{\rho} + 3\gamma H \rho = 0. \quad (28)$$

By comparing the above equations with the ones we considered in the previous section for the CET, it is easy to realize that the difference is the quadratic effect due to the energy density  $\rho$ . Moreover, the brane cosmology will reduce to CET if we take the limit  $k_5 \rightarrow 0$  and adjust the value of  $k_4$  accordingly. We will perform the stability analysis, in the next section, for the brane universe. The CET can be reproduced by imposing the limit  $k_5 \rightarrow 0$ .

The general solution of the above system was obtained in the exact parametric form in [14] by the same approach used in the previous section. Instead of expressing results as functions of time, we are able to present all variables,

including time, in terms of volume scale factor,  $V$ , with  $V \geq 0$ . For instance, the time variable can be expressed as

$$t - t_0 = \int F(V)^{-1/2} dV, \quad (29)$$

where  $F(V)$  is defined as

$$F(V) \equiv 3\Lambda V^2 + 3k_4^2 \rho_0 V^{2-\gamma} + \frac{1}{4} k_5^4 \rho_0^2 V^{2-2\gamma} + C, \quad (30)$$

with  $\rho_0$  and  $C$  the constants of integration. The other variables are

$$H = \frac{F(V)^{1/2}}{3V}, \quad (31)$$

$$a_i = a_{0i} V^{1/3} \exp \left[ K_i \int V^{-1} F(V)^{-1/2} dV \right], \quad i = 1, 2, 3, \quad (32)$$

$$A = 3K^2 F(V)^{-1}, \quad (33)$$

$$q = \frac{3\gamma(6k_4^2 \rho_0 V^{2-\gamma} + k_5^4 \rho_0^2 V^{2-2\gamma}) + 12C}{4F(V)} - 1, \quad (34)$$

where  $a_{0i}$ ,  $K_i$ ,  $i = 1, 2, 3$  are constants of integration and  $K^2 = \sum_{i=1}^3 K_i^2$ . In addition, the arbitrary integration constants  $K_i$  and  $C$  must satisfy the same consistency condition  $K^2 = 2C/3$ .

One can immediately show that the effect of the energy density quadratic term becomes significant at the high energy epoch, or in another words, at the early stages of the Universe by looking at the Eq. (30). Indeed,  $F(V) \propto V^{2-2\gamma}$  at the limit  $t \rightarrow 0$  when  $V$  is extremely small. As a result, the  $F$  diverges as  $t \rightarrow 0$ . Therefore, the mean anisotropy parameter  $A \rightarrow 0$  at the early universe. On the other hand, in the large time limit, the properties of the Universe should be more or less the same as the case we have discussed for the CET in the previous section by looking at the same Eq. (30). Hence the early evolution of the anisotropic Bianchi type I brane Universe is dramatically changed due to the presence of the brane correction terms proportional to the square of the energy density. The time variation of the mean anisotropy parameter of the Bianchi type I space-time is presented, for different values of  $\gamma$ , in Fig.3. From the Fig.3, it is clear that at high energy density the evolution of the brane Universe always starts out from an isotropic state with  $A \rightarrow 0$ . The mean anisotropy parameter increases and reaches a maximum value after a finite time interval  $t_c$ . One can show that, when  $t > t_c$ , the mean anisotropy parameter is a monotonically decreasing function approaching zero in the large time limit. This behavior is in sharp contrast to the usual evolution in the CET, as shown in Fig.1, in which the Universe has to kick off from state of maximum anisotropy due to the constraint from the field equation.

In addition, the early time evolution of the brane universe is normally not in an inflationary phase. On the other hand, the brane Universe always ends up in an inflationary phase in the large time limit in the presence of a nonvanishing cosmological constant. These are generic features of the brane Universe due to the constraint of the field equations on the brane cosmology. Indeed, a more detailed information can be extracted from the Eq. (29) in the limit  $t \rightarrow 0$ , or equivalently, the case with a small  $V$ . For simplicity, one will take  $C = 0$  again. Indeed, we can show that  $V \propto t^{1/\gamma}$  as  $t \rightarrow 0$ . Hence, the expansion of the early universe is of the form of power law expansion. In addition, in the early stages of evolution of the brane Universe the mean anisotropy parameter varies as  $A \propto t^{2-2/\gamma}$  approaching zero as  $t \sim 0$ . Moreover, the deceleration parameter is given by  $q = 3\gamma - 1$  which is always positive for all possible values of  $\gamma$  for the case  $C = 0$ .

### C. Stability Analysis

The general perturbations for the FRW background with perfect fluid can be found in Ref. [16]. The same consideration is, however, more complicate for the anisotropic background. For the primary effect, we will only consider the scalar mode and neglect the vector and tensor modes [18, 19]. The metric perturbation considered here is

$$a_i \rightarrow a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i), \quad (35)$$

and the perturbations with respect to the perfect fluid considered in this paper is

$$\rho \rightarrow \rho_B + \delta \rho, \quad p \rightarrow p_B + (\gamma - 1)\delta \rho. \quad (36)$$

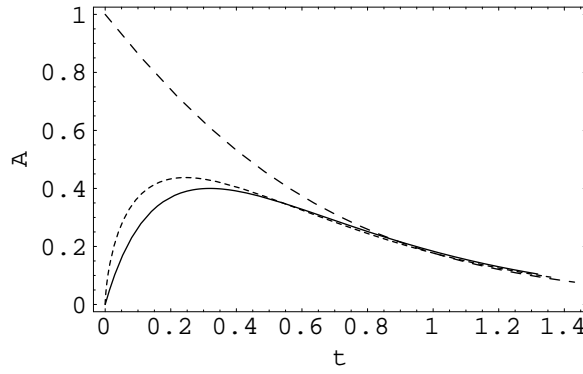


FIG. 3: Mean anisotropy parameter of the Bianchi type I brane Universe with a perfect fluid:  $\gamma = 2$  (solid curve),  $\gamma = 1.5$  (dotted curve) and  $\gamma = 1$  (dashed curve). The normalization of the parameters is chosen as  $3\Lambda = 1$ ,  $3k_4^2\rho_0 = 2$ ,  $k_5^4\rho_0^2 = 4$ , and  $C = 1$ .

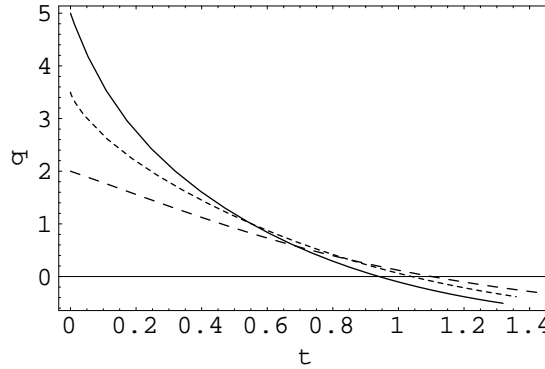


FIG. 4: Deceleration parameter of Bianchi type I brane universe with a perfect fluid:  $\gamma = 2$  (solid curve),  $\gamma = 1.5$  (dotted curve) and  $\gamma = 1$  (dashed curve). The normalization of the parameters is chosen as  $3\Lambda = 1$ ,  $3k_4^2\rho_0 = 2$ ,  $k_5^4\rho_0^2 = 4$ , and  $C = 1$ .

Here the variables with subscript  $B$  are the exact solutions presented in the previous sections. For technical convenience, we will use variables  $\delta b_i$  instead of  $\delta a_i$  in our analysis. Therefore, the perturbations of the following quantities can be shown to be

$$H_i \rightarrow H_{Bi} + \delta \dot{b}_i, \quad H \rightarrow H_B + \frac{1}{3} \sum_i \delta \dot{b}_i, \quad \sum_i H_i^2 \rightarrow \sum_i H_{Bi}^2 + 2 \sum_i H_{Bi} \delta \dot{b}_i, \quad V \rightarrow V_B + V_B \sum_i \delta b_i. \quad (37)$$

As discussed in the previous sections, the results in the CET are special cases for the brane universe, in the limit  $k_5 = 0$ . Therefore, we can perform our stability analysis only for the brane universe first. The conclusion for the CET can be easily extracted from the brane world case.

The perturbation equations for various quantities, can be obtained by substituting the perturbations (36, 37) into the field equations (26, 27, 28). Leading terms will reproduce the field equations. Therefore, one has the following perturbation equations from the first order terms  $O(\delta b_i, \delta \rho)$ ,

$$\sum_{i=1}^3 \delta \ddot{b}_i + 2 \sum_{i=1}^3 H_{Bi} \delta \dot{b}_i = -\frac{3\gamma-2}{2} k_4^2 \delta \rho - \frac{3\gamma-1}{6} k_5^4 \rho_B \delta \rho, \quad (38)$$

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta \dot{b}_i + H_{Bi} \sum_{j=1}^3 \delta \dot{b}_j = -\frac{\gamma-2}{2} k_4^2 \delta \rho - \frac{\gamma-1}{6} k_5^4 \rho_B \delta \rho, \quad i = 1, 2, 3, \quad (39)$$

$$\delta \dot{\rho} + 3\gamma H_B \delta \rho + \gamma \sum_{i=1}^3 \delta \dot{b}_i \rho_B = 0. \quad (40)$$

In order to solve the above system of differential equations, we need an inspiration from the process of constructing the exact solutions. First of all, from the general results (15, 17) in both the CET and the brane theory, their dynamics

on the perturbation variables follow the constraints

$$\delta\rho = -\gamma\rho_B \sum_{i=1}^3 \delta b_i, \quad (41)$$

$$\delta\dot{b}_i = \frac{1}{3} \sum_{j=1}^3 \delta\dot{b}_j - \frac{K_i}{V_B} \sum_{j=1}^3 \dot{b}_j. \quad (42)$$

By summing the Eqs. (39) and then using the result (41), we end up with a second order differential equation for a variable of the combination  $\sum_{i=1}^3 \delta b_i$

$$\sum_{i=1}^3 \delta\ddot{b}_i + 6H_B \sum_{i=1}^3 \delta\dot{b}_i - \frac{1}{2}\gamma\rho_B[3(\gamma-2)k_4^2 + (\gamma-1)k_5^4\rho_B] \sum_{i=1}^3 \delta b_i = 0. \quad (43)$$

The task is to solve  $\sum \delta b_i$  from the above equation and then to construct  $\delta b_i$  and  $\delta\rho$  from the Eqs. (42) and (41). For the purpose of the stability analysis in the final stage, it is sufficient to consider the large time limit behaviors of the perturbation variables  $\delta b_i$  and  $\delta\rho$ . From the discussion on the asymptotic behavior of the exact solutions, qualitative outcome in both the CET and the brane theory, we should divide our analysis into two different cases in the presence of absence of the cosmological constant.

For the case with a positive cosmological constant, one can extract the asymptotic forms of the background variables from the exact solutions which give

$$H_B \rightarrow \sqrt{\Lambda/3}, \quad V_B \propto \exp(\sqrt{3\Lambda}t), \quad \rho_B \propto \exp(-\sqrt{3\Lambda}\gamma t). \quad (44)$$

The asymptotic expression of  $\rho_B$  indicates that the third term in Eq. (43) can be neglected in the large time limit. As a result, we have the following equation for the asymptotic  $\sum_{i=1}^3 \delta b_i$  which remains valid in both the CET and the brane world

$$\sum_{i=1}^3 \delta\ddot{b}_i + \sqrt{12\Lambda} \sum_{i=1}^3 \delta\dot{b}_i = 0. \quad (45)$$

This in term leads to the final result

$$\sum_{i=1}^3 \delta b_i \propto \exp(-\sqrt{12\Lambda}t). \quad (46)$$

Therefore, from Eqs. (41, 42) we can obtain the asymptotic expressions of the following perturbation variables

$$\delta b_i \propto \exp(-\sqrt{12\Lambda}t), \quad \delta\rho \propto \exp[-\sqrt{3\Lambda}(\gamma+2)t]. \quad (47)$$

This indicates that the background solutions are stable.

Similarly, for the case with  $\Lambda = 0$ , we have

$$H_B \rightarrow \frac{2}{3\gamma}t^{-1}, \quad V_B \rightarrow \left( \frac{\sqrt{3\rho_0 k_4^2 \gamma}}{2} t \right)^{2/\gamma}, \quad \rho_B \rightarrow \frac{4}{3k_4^2 \gamma^2} t^{-2}, \quad (48)$$

where the preceding coefficients become important in this case. Therefore, the Eq. (43), when the brane correction due to the quadratic energy density term signified by the parameter  $k_5$  can be neglected in the large time limit, reduces to

$$\sum_{i=1}^3 \delta\ddot{b}_i + \frac{4}{\gamma}t^{-1} \sum_{i=1}^3 \delta\dot{b}_i - \frac{2(\gamma-2)}{\gamma}t^{-2} \sum_{i=1}^3 \delta b_i = 0. \quad (49)$$

The solution of the above equation has the form

$$\sum_{i=1}^3 \delta b_i \propto t^\alpha, \quad (50)$$



where the exponent parameter is determined by

$$\alpha(\alpha - 1) + \frac{4}{\gamma}\alpha - \frac{2(\gamma - 2)}{\gamma} = 0. \quad (51)$$

The explicit expression of  $\alpha$  is

$$\alpha = \frac{(\gamma - 4) \pm \sqrt{(4 - \gamma)^2 + 8\gamma(\gamma - 2)}}{2\gamma} \leq 0, \quad (52)$$

which is always negative for all possible values of  $\gamma$ . The only exception is the case when  $\gamma = 2$  such that  $\alpha = 0$  is a possible solution. In summary, the asymptotic behavior of perturbation variables  $\delta b_i$  and  $\delta \rho$  is

$$\delta b_i \propto t^\alpha, \quad \delta \rho \propto t^{\alpha-2}, \quad (53)$$

which shows that the background solutions are always stable even when the cosmological constant is vanishing.

#### IV. ANISOTROPIC UNIVERSE WITH SCALAR FIELD

In the previous section we have considered the evolution of the universe with a perfect fluid, obeying a barotropic equation of state, in the anisotropic background. We will study the stability problem of the anisotropic universe with a scalar field in this section. It is known that the scalar field  $\phi$  with a potential can be thought of as a perfect fluid with the energy density and pressure given by  $\rho_\phi = \dot{\phi}^2/2 + U(\phi)$  and  $p_\phi = \dot{\phi}^2/2 - U(\phi)$  respectively. Here  $U(\phi)$  is the scalar field potential. The scalar field is expected to play a fundamental role in the evolution of the early Universe.

In this section, we will only consider the case where the scalar field potential is a constant,  $U(\phi) = \Lambda = \text{constant} > 0$  acting as a cosmological constant. As discussed earlier, the related anisotropic cosmology in the CET is just a particular limit of the brane universe. Therefore, we will consider the brane universe in details for the moment. The CET can be recovered by taking a suitable limit.

When we discuss the stability problem of the system, we will focuss on the system with a scalar potential that admits at least a local minimum  $\phi_0$  such that  $U(\phi = \phi_0) = \Lambda$ . We will assume that  $\phi_0$  is the asymptotic solution to the field equation in the large time limit. We will able to show that the system remains stable in the large time limit even the evolutionary solution is only known when the scalar potential is a constant.

##### A. Brane Cosmology

For a Bianchi type I brane Universe with a scalar field the gravitational field equations take the following form [14]:

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = k_4^2 [U(\phi) - \dot{\phi}^2] - k_5^4 \left[ \frac{1}{6} \left( \frac{\dot{\phi}^2}{2} + U(\phi) \right)^2 + \frac{1}{4} \left( \frac{\dot{\phi}^4}{4} - U^2(\phi) \right) \right], \quad (54)$$

$$\frac{1}{V} \frac{d}{dt} (V H_i) = k_4^2 U(\phi) - \frac{1}{12} k_5^4 \left( \frac{\dot{\phi}^4}{4} - U^2(\phi) \right), \quad i = 1, 2, 3. \quad (55)$$

Note that the scalar field also obeys the following evolution equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dU(\phi)}{d\phi} = 0. \quad (56)$$

By imposing the constraint  $U(\phi) = \Lambda$ , the Eq. (56) gives  $\dot{\phi} = 2\phi_0/V$  with  $\phi_0 > 0$  a constant of integration. Summing the Eq. (55) over the index  $i$  and comparing with the Eq. (54), one can extract the following equation:

$$\ddot{V} = \kappa^2 V - \kappa_0^2 V^{-3}. \quad (57)$$

Here we have denoted the positive constants  $\kappa$  and  $\kappa_0$  as  $\kappa^2 = 3k_4^2\Lambda + k_5^4\Lambda^2/4$  and  $\kappa_0^2 = k_5^4\phi_0^4$ .

The general solution of the Eq. (57) can be shown to be [14]

$$V(t) = \frac{1}{2\kappa} e^{-\kappa(t-t_0)} \sqrt{[e^{2\kappa(t-t_0)} - C]^2 - 4\kappa_0^2\kappa^2}. \quad (58)$$

Here  $C$  is an integration constant.

The time evolution of the expansion, scale factors, mean anisotropy, shear and deceleration parameter are given by

$$H(t) = \frac{\kappa}{3} \frac{e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2 \kappa^2}{(e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2 \kappa^2}, \quad (59)$$

$$a_i(t) = a_{0i} e^{-\kappa(t-t_0)/3} \left[ (e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2 \kappa^2 \right]^{1/6} \exp \left[ 2K_i F(e^{\kappa(t-t_0)}) \right], \quad i = 1, 2, 3, \quad (60)$$

$$A(t) = \frac{12K^2 e^{2\kappa(t-t_0)} \left[ (e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2 \kappa^2 \right]}{[e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2 \kappa^2]^2}, \quad (61)$$

$$q(t) = 12e^{2\kappa(t-t_0)} \frac{C e^{4\kappa(t-t_0)} + (C^2 - 4\kappa_0^2 \kappa^2) [C - 2e^{2\kappa(t-t_0)}]}{[e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2 \kappa^2]^2} - 1, \quad (62)$$

where  $a_{0i}$ ,  $i = 1, 2, 3$  are arbitrary constants of integration and  $F(x) = \int [(x^2 - C)^2 - 4\kappa_0^2 \kappa^2]^{-1/2} dx$ . We do expect that the Universe starts its evolution from a singular initial condition. Therefore, according to the value  $V(0)$ , the parameter  $t_0$  should be chosen as  $\exp(-2\kappa t_0) = C + 2\kappa_0 \kappa$ .

Note that we can analyze the evolution of the Universe in two different regions. First of all, at the very early time, the parameters can be shown to be

$$V \sim \sqrt{2\kappa_0 t} \sim 0, \quad A \sim 6K^2 t / \kappa_0 \sim 0, \quad q \sim 5 > 0. \quad (63)$$

Secondly, the asymptotic behavior of the parameters can be shown to be

$$V \propto e^{\kappa t}, \quad A \propto e^{-2\kappa t}, \quad q \rightarrow -1 < 0. \quad (64)$$

This result is similar to the model with a perfect fluid such that the brane Universe both evolves from an isotropic singular state initially. Later on, the mean anisotropy parameter  $A$  increases dynamically and decays to zero after the mean anisotropy parameter reaches its maximal value.

## B. Conventional Einstein's Theory

The conventional Einstein's theory (CET) can be easily deduced from the above results by taking the limit  $k_5 = 0$ , or, in this case,  $\kappa_0 = 0$ . The result is

$$V(t) = \frac{\sqrt{C}}{2\kappa} (e^{\kappa t} - e^{-\kappa t}), \quad (65)$$

$$H(t) = \frac{\kappa}{3} \frac{e^{2\kappa t} + 1}{e^{2\kappa t} - 1}, \quad (66)$$

$$a_i(t) = a_{0i} C^{1/6} (e^{\kappa t} - e^{-\kappa t})^{1/3} \exp \left[ 2K_i C^{-1} \int (e^{2\kappa x} - 1)^{-1} dx \right], \quad i = 1, 2, 3, \quad (67)$$

$$A(t) = 12K^2 C^{-1} (e^{\kappa t} + e^{-\kappa t})^{-2} \quad (68)$$

$$q(t) = 12 (e^{\kappa t} + e^{-\kappa t})^{-2} - 1. \quad (69)$$

Note that  $\exp(-2\kappa t_0) = C$ . The asymptotic behavior is the same as the case in the brane world. The initial state is, however, very different such that

$$V \sim \sqrt{C} t \sim 0, \quad A \sim 3K^2 / C \sim \text{constant}, \quad q \sim 2 > 0. \quad (70)$$

Therefore, for the CET, the Universe has to start out from an anisotropic initial expansion and then decays all the way to the phase of isotropic expansion in the large time limit.

The evolution of the mean anisotropy parameter  $A(t)$  for the CET and the brane world is presented in the Fig.5.

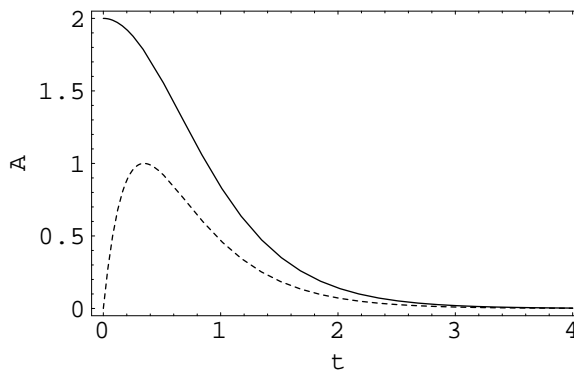


FIG. 5: Mean anisotropy parameter for the Bianchi type I universe with a scalar field: for the CET (solid curve) and for the brane world (dotted curve). The normalization of the parameters is set as  $\kappa = 1, 2\kappa_0\kappa = C$ .

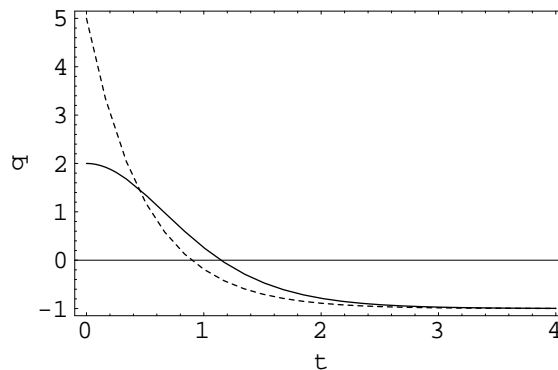


FIG. 6: Deceleration parameter for the Bianchi type I universe with a scalar field: for the CET (solid curve) and for the brane world (dotted curve). The normalization of the parameters is set as  $\kappa = 1, 2\kappa_0\kappa = C$ .

### C. Stability Analysis

Since the quadratic brane correction plays a significant role only at the early stage of the evolution of the Universe. For the purpose of our stability analysis in the large time limit, we will focus on the asymptotic behavior of the fields. Therefore, it is sufficient to consider the background solutions in the conventional Einstein's theory.

Perturbations of the fields of a gravitational system against the background evolutionary solution should be checked to ensure the stability of the exact or approximated background solution. In principle, the stability analysis should be performed against the perturbations of all possible fields in all possible manners subject to the field equations and boundary conditions of the system. In the following section, we will divide the perturbations into two disjoint classes: (a) the perturbations of the scale factors, or equivalently the metric field; and (b) the perturbations of the dilaton field.

We will argue that the most complete stability conditions we are looking for can be obtained from class (a) and class (b) perturbations; even the backreaction of the scalar field perturbation on the metric field perturbations is known to be important [25]. We will show that this backreaction does not bring in any further restriction on the stability conditions.

The reason is rather straightforward. One can write the linearized perturbation equation as

$$D_{a_j}^i \delta a_j + D_\phi^i \delta \phi = 0, \quad (71)$$

for the system we are interested. Moreover, perturbations are defined as  $a_i = a_i^0 + \delta a_i$  and  $\phi = \phi_0 + \delta \phi$  with the index 0 denoting the background field solution. Note also that the operators  $D_{a_j}^i$  and  $D_\phi^i$  denote the differential operator one obtained from the linearized perturbation equation with all fields evaluated at the background solutions. The exact form of these differential operators will be shown later in the following arguments.

One is looking for stability conditions that the field parameters must obey in order to keep the evolutionary solution stable. One can show that class (a) and class (b) solutions are good enough to cover all domain of stability

conditions. Let us denote the domain of solutions to class (a), (b), and (a+b) stability conditions as  $S(a)$ ,  $S(b)$ , and  $S(a+b)$  respectively. Specifically, the definition of these domains are defined by  $S(a) \equiv \{\delta a_i | D_{a_j}^i \delta a_j = 0\}$ ,  $S(b) \equiv \{\delta\phi | D_\phi^i \delta\phi = 0\}$ , and  $S(a+b) \equiv \{(\delta a_i, \delta\phi) | D_{a_j}^i \delta a_j + D_\phi^i \delta\phi = 0\}$ .

Therefore, one only needs to show that  $S(a) \cap S(b) \subset S(a+b)$ . This is because that “ $D_{a_j}^i \delta a_j = 0$  and  $D_\phi^i \delta\phi = 0$ ” imply that “ $D_{a_j}^i \delta a_j + D_\phi^i \delta\phi = 0$ .” On the other hand, “ $D_{a_j}^i \delta a_j + D_\phi^i \delta\phi = 0$ ” does not imply that “ $D_{a_j}^i \delta a_j = 0$  or  $D_\phi^i \delta\phi = 0$ .” Hence class (a) and class (b) solutions cover all the required stability conditions we are looking for. Hence we only need to consider these two separate cases for simplicity.

In addition, one knows that any small time-dependent perturbation against the metric field is known to be equivalent to a gauge choice [26]. This can be clarified as follows. Indeed, one can show that any small coordinate change of the form  $x'^\mu = x^\mu - \epsilon^\mu$  will induce a gauge transformation on the metric field according to  $g'_{\mu\nu} = g_{\mu\nu} + D_\mu \epsilon_\nu + D_\nu \epsilon_\mu$ . Therefore, a small metric perturbation against a background metric is amount to a gauge transformation of the form  $a'_i = a_i + \epsilon^t \dot{a}_i$  for the Bianchi type-I metric with  $\epsilon_\mu = (\text{constant}, \epsilon_i(t))$ . This is then equivalent to small metric perturbations. If a background solution is stable against small perturbation with respect to small field perturbations, one in fact did nothing but a field redefinition.

If the background solution is, however, unstable against small perturbations, e.g., the small perturbation will grow exponentially as we will show momentarily, the resulting large perturbations cannot be classified as small gauge transformation any more. Therefore, the stability analysis performed in the literature [10, 11, 20, 21, 22, 23, 24, 28, 29] for various models against the unstable background solution served as a very simple method to check if the system supports a stable metric field background. This is the reason why we still perform a perturbation on the metric field for stability analysis; even a small perturbation is equivalent to a gauge redefinition.

Note that one should also consider a more general perturbation with space perturbation included. The formulation is, however, much more complicated than the one we will show in this paper. We will focus on the time-dependent case for simplicity in this paper. The space-dependent perturbation analysis is still under investigation. The time-dependent analysis alone will, however, bring us much useful information for the stability conditions about the model we are interested. For example, we will show in the following subsection that the solution found in Ref. [14] remains stable as long as the scalar field falls close to any local minimum of the potential  $U(\phi)$ . Note again that the solution found in Ref. [14] is an exact solution only when  $U = \text{constant}$ .

### Dilaton Perturbation

Let us consider the perturbation of the dilaton field of the following form

$$\phi \rightarrow \phi_B + \delta\phi, \quad U(\phi) \rightarrow \Lambda + \partial_\phi \delta\phi. \quad (72)$$

By setting  $k_5 = 0$  in the Eqs. (54, 55, 56), the field equations for  $\delta\phi$  become

$$-2\dot{\phi}_B \delta\dot{\phi} + \partial_\phi U \delta\phi = 0, \quad (73)$$

$$\partial_\phi U \delta\phi = 0, \quad (74)$$

$$\delta\ddot{\phi} + 3H_B \delta\dot{\phi} + \partial_\phi^2 U \delta\phi = 0. \quad (75)$$

Due to the fact that  $H_{Bi} \rightarrow \frac{\kappa}{3}$ , the asymptotic solution of  $\delta\phi$  can be shown to be

$$\delta\phi \propto \exp \left[ (-\kappa \pm \sqrt{\kappa^2 - 4\beta})t/2 \right]. \quad (76)$$

Here  $\beta \equiv \partial_\phi^2 U|_{t \rightarrow \infty}$ . Therefore, the perturbation of dilaton field decays to zero if  $\beta \geq 0$ . Hence we show that the system is stable with respect to the scalar perturbation if  $\beta \geq 0$  such that the scalar potential is capable of confining the scalar field to its local minimum.

Note that there are two additional equation  $U' = 0$  and  $\dot{\phi} = 0$  which is required for consistency of the stability of the system. These constraints simply imply that the scalar field must be at rest at the local minimum of the scalar field potential. As we have pointed out earlier that the background solution we have at hand is an exact solution for the model with a constant cosmological constant. Therefore, the background solution we used for stability analysis is only an approximated solution which remains valid only when the scalar field is close to the local minimum of the the scalar field potential. As a result, one should not be too serious about these two further constraints. In fact, these constraints are both negligible in the large time limit when the scalar field falls close to the local minimum of the scalar field potential, namely,  $U' \rightarrow 0$  and  $\dot{\phi} \rightarrow 0$ . Hence one is able to show that these additional constraints can be made satisfied approximately in the large time limit.

### Metric Perturbation

Using the metric perturbation (37), the perturbation equations for the metric perturbation  $\delta b_i$  can be obtained from perturbing the Eqs. (54, 55, 56). The result is

$$\sum_i \ddot{\delta b}_i + 2 \sum_i H_{Bi} \dot{\delta b}_i = 0, \quad (77)$$

$$\ddot{\delta b}_i + \frac{\dot{V}_B}{V_B} \dot{\delta b}_i + H_{Bi} \sum_j \dot{\delta b}_j = 0, \quad (78)$$

$$\dot{\phi}_B \sum_i \dot{\delta b}_i = 0. \quad (79)$$

Here we also choose the limit  $k_5 = 0$ . Note that the background variables  $V_B$ ,  $a_{Bi}$  and  $H_{Bi}$  approach

$$V_B \propto e^{\kappa t}, \quad a_{Bi} \propto e^{\kappa t/3}, \quad H_{Bi} \rightarrow \frac{\kappa}{3}. \quad (80)$$

Therefore, the asymptotic behavior of the metric perturbation  $\delta b_i$  can be found from the second equation. In addition, the other two field equations will provide a constraint for the system. The results are

$$\delta b_i \rightarrow c_i e^{-\kappa t}, \quad \sum_i c_i = 0. \quad (81)$$

Hence, the metric perturbation

$$\delta a_i \equiv a_{Bi} \delta b_i \rightarrow e^{-2\kappa t/3} \rightarrow 0, \quad (82)$$

in the large time limit. This indicates that the background solution of the system is stable against the metric perturbation as shown above.

## V. CONCLUSION

We have discussed the anisotropic property of cosmological models in the CET and brane theory. A realistic model, being consistent with the current observations, should produce a small value of anisotropic parameter at the later stage of the evolution of our Universe near the last scattering surface. By assuming the Bianchi type I space-time for the evolution of our Universe, we found that the final state of the evolving Universe always approaches the phase of isotropic expansion in both theories.

These two different theories give completely different initial anisotropy at the very early stage of evolution. Indeed, for the CET, the anisotropy tends to be large in the very early stage. In another words, the universe tends to begin from a highly anisotropic initial state. The mean anisotropy parameter  $A$  will then decay to zero as the time increases. On the other hand, the early time behavior of the Universe in the brane world scenario changes significantly due to the quadratic correction on the brane. As a result, any non-vanishing mean anisotropy parameter,  $A(t)$ , tends to vanish in the very early period. There is a characteristic time,  $t_c$ , that divides the evolution of  $A(t)$  into two different stages. The mean anisotropy parameter is increasing when  $t < t_c$  and reaches its maximal value at  $t = t_c$ . After that,  $A(t)$  starts to decay. This kind of behavior is clearly shown in the Fig.2 and Fig.5. This result remains true for both the model with a perfect fluid and the model with a scalar field. And this appears to be a general feature independent of the types of matter considered.

It is worth noting that the only exception is the model with  $p = 0$  (i.e.  $\gamma = 1$ ) of the perfect fluid model. The mean anisotropy of this model behaves similar to the models in the CET where mean anisotropy parameter is large in the very early time. Moreover, we also analyzed the stability problem for those exactly solved anisotropic models shown in this paper. The result indicates that all of the solution known to us are stable in the large time limit. Therefore, the evolution of the Universe in the CET starts with highly initial anisotropic expansion. The dynamic of the system will take the Universe to the phase of isotropic expansion in the large time limit. We also show that the final isotropic expansion will remain stable in the large time limit. In addition, the mean anisotropy parameter will keep decreasing as time increases. The model provided here is a useful and explicit model that is capable of providing us with a Universe that has a tiny anisotropy left over near the last scattering surface.

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- [1] S. Gulkis, P. M. Lubin, S. S. Meyer and R. F. Silverberg, *The cosmic background explorer*, *Sci. Am.* **262**(1) 122-129 (1990).
  - [2] D. J. Fixsen, *et al*, *The cosmic microwave background spectrum from the full COBE firas data set*, *Astrophys. J.* **473** (1996) 576-587.
  - [3] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83**, 3370-3373 (1999); hep-ph/9905221.
  - [4] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett* **83**, 4690-4693 (1999); hep-th/9906064.
  - [5] C. W. Misner, *Mixmaster universe*, *Phys. Rev. Lett.* **22** (1969) 1071-1074.
  - [6] E. M. Lifshitz and I. M. Khalatnikov, *Problems of relativistic cosmology*, *Sov. Phys. Uspekhi* **6** (1963) 495-522. I. M. Khalatnikov and E. M. Lifshitz, *General cosmological solution of the gravitational equations with singularity in time*, *Phys. Rev. Lett.* **24** (1970) 76-79.
  - [7] J. D. Barrow, J. Stein-Schabes, *Kaluza-Klein mixmaster universes*, *Phys. Rev.* **D32** (1985) 1595-1596. J. D. Barrow, K. E. Kunze, *Inhomogeneous string cosmologies*, *Phys. Rev.* **D56** (1997) 741-752; hep-th/9701085.
  - [8] N. J. Cornish and J. J. Levin *The mixmaster universe: a chaotic farey tale*, *Phys. Rev.* **D55** (1996) 7489-7510, gr-qc/9612066. *The mixmaster universe is chaotic*, *Phys. Rev. Lett.* **78** (1997) 998-1001, gr-qc/9605029.
  - [9] M. Weaver, J. Isenberg and B. K. Berger, *Mixmaster behavior in inhomogeneous cosmological space-times*, *Phys. Rev. Lett.* **80** (1998) 2984-2987; gr-qc/9712055.
  - [10] J. D. Barrow and A. C. Ottewill, *The stability of general relativistic cosmological theory* *J. Phys. A: Math. Gen.* **16** (1983) 2757-2776.
  - [11] M. S. Madsen and J. D. Barrow, *De Sitter ground states and boundary terms in generalized gravity*, *Nucl. Phys.* **B323** (1989) 242-252.
  - [12] J. D. Barrow and H. Kodama, *The isotropy of compact universes*, gr-qc/0012075.
  - [13] T. Shiromizu, K. Maeda and M. Sasaki, *The Einstein equation on the 3-brane world*, *Phys. Rev.* **D62**, 024012 (2000); gr-qc/9910076.
  - [14] C.-M. Chen, T. Harko and M. K. Mak, *Exact anisotropic brane cosmologies*, *Phys. Rev.* **D64** (2001) 044013; hep-th/0103240.
  - [15] M. Sasaki, T. Shiromizu and K. Maeda, *Gravity, stability and energy conservation on the Randall-Sundrum brane world*, *Phys. Rev.* **D62**, 024008 (2000); hep-th/9912233.
  - [16] D. Langlois, R. Maartens, M. Sasaki and D. Wands, *Large scale cosmological perturbations on the brane*, *Phys. Rev.* **D63** (2001) 084009; hep-th/0012044.
  - [17] C.-M. Chen, T. Harko and M. K. Mak, *Anisotropic four-dimensional Neveu-Schwarz-Neveu-Schwarz string cosmology*, *Phys. Rev.* **D63** (2001) 064002; hep-th/0005236.
  - [18] C.-M. Chen and W. F. Kao, *Stability analysis of anisotropic inflationary cosmology*, *Phys. Rev.* **D64** (2001) 124019; hep-th/0104101.
  - [19] W. F. Kao, *Bianchi type I space and the stability of inflationary Friedmann-Robertson-Walker space*, *Phys. Rev.* **D64** (2001) 107301; hep-th/0104166.
  - [20] W. F. Kao and U.-L. Pen, *Generalized Friedmann-Robertson-Walker metric and redundancy in the generalized Einstein Equations*, *Phys. Rev.* **D44** (1991) 3974-3977.
  - [21] A. Dobado and A. L. Maroto, *Inflatonless inflation*, *Phys. Rev.* **D52** (1995) 1895-1901, Erratum: *ibid.* **D53** (1996) 2262; hep-ph/9406409.
  - [22] A. L. Maroto and I. L. Shapiro, *On the inflationary solutions in higher-derivative gravity with dilaton field*, *Phys. Lett.* **B414** (1997) 34-44; hep-th/9706179.
  - [23] W. F. Kao, U.-L. Pen and P. Zhang, *Friedmann equation and stability of inflationary higher derivative gravity*, *Phys. Rev.* **D63** (2001) 127301; gr-qc/9911116.
  - [24] W. F. Kao, *Inflationary universe in higher derivative induced gravity*, *Phys. Rev.* **D62** (2000) 087301; hep-th/0003206.
  - [25] V. F. Mukhanova, H. A. Feldman and R. H. Brandenberger, *Theory of cosmological perturbations*, *Phys. Rept.* **215** (1992) 203-333.
  - [26] W. Unruh, *Cosmological long wavelength perturbations*, astro-ph/9802323.
  - [27] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven-dimensions*, *Nucl. Phys.* **B460** (1996) 506-524; hep-th/9510209.
  - [28] S.-W. Kim and H. Lee, *Exact solutions of charged wormhole*, *Phys. Rev.* **D63** (2001) 064014; gr-qc/0102077.
  - [29] S. A. Abel and G. Servant, *Dilaton stabilization in effective type I string models*, *Nucl. Phys.* **B597** (2001) 3-22; hep-th/0009089.